

# Interest Rate Determination in the Interbank Market

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## Abstract

The purpose of this paper is to study the determinants of equilibrium in the market for daily funds. We use the EONIA panel database which includes daily information on the lending rates applied by contributing commercial banks. The data clearly shows an increase in both the time series volatility and the cross section dispersion of rates towards the end of the reserve maintenance period. These increases are highly correlated. With respect to quantities, we find that the volume of trade as well as the use of the standing facilities are also larger at the end of the maintenance period. Our theoretical model shows how the operational framework of monetary policy causes a reduction in the elasticity of the supply of funds by banks throughout the reserve maintenance period. This reduction in the elasticity together with market segmentation and heterogeneity are able to generate distributions for the interest rates and quantities traded with the same properties as in the data.

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## 1. INTRODUCTION

The purpose of this paper is to study the determinants of equilibrium in the market for daily funds. Understanding the behaviour of the overnight market for unsecured loans is important both from a policy as well as from a research point of view. The basic reason is that this market hosts the first step in the monetary transmission mechanism. There is a long literature analysing this mechanism, that is, the process by which central banks are able to affect their ultimate goals of policy through changes in policy instruments under this context. However, most of the papers in this literature simplify matters by assuming central banks have direct control of a short-term interest rate (see, for example, Taylor (1999) or McCallum (1999)). Here, we construct a theoretical model of the money market to explicitly analyse *how* this control is actually exerted.

There are several issues we address with this model. First, we look at the linkages between the statistical properties of the equilibrium in the overnight market and the operational framework of monetary policy. We see this as a necessary step in order to address questions about the effects of changes in the design of the central banks' operational framework. Relevant features include reserve requirements, length of reserve maintenance periods, existence of standing facilities, maturities and frequency of open market operations, etc. Second, although the model cannot be estimated directly, we use a set of testable implications to take it to the data. It turns out that the model is able to reproduce the most salient features that characterise the overnight interest rates in the euro area.

Most available empirical studies on the high frequency behaviour of overnight interest rates focus on the US case. References include Campbell (1987), Lasser (1992), Rudebusch (1995), Roberds *et al.* (1996), Hamilton (1996), Balduzzi *et al.* (1997), Furfine (2000) and Bartolini, Bertola and Prati (2001, 2002). Prati, Bertola and Bartolini (2002) argue that some of the empirical facts identified for the US are no longer relevant when alternative institutional settings are considered. The case of a "corridor system" is particularly relevant. In a corridor system overnight market interest rates are bound by the existence of two standing facilities provided by the central bank, with pre-determined interest rates. A deposit facility where banks can deposit their excess clearance balances, earning a given return and a lending facility which provides access to liquidity, at a given interest rate, against the pledging of eligible collateral. Outside the US the "corridor system" has been adopted by a series of countries during the last decade, namely Australia, Canada, Denmark, the euro area, New Zealand, Sweden, and the UK. Since the start of the ECB's single monetary policy in 1999, a significant amount of research has been devoted to identifying the relevant empirical facts characterising the euro's market for overnight funds. Relevant references include Angeloni and Bisagni (2002), Cassola and Morana (2002), and Würtz (2003). In this paper we pursue this empirical research programme further by considering lending rates charged by individual banks. To our knowledge, this is the first paper in the

literature to address the determination of rates from this point of view, particularly focused in the Euro area. Specifically, we use data on individual banks to study the joint statistical distribution of overnight rates over time and over the cross section of banks.

In our research we have been able to use the EONIA panel database, kindly made available by the European Banking Federation (EBF). This database includes daily information on the lending rates applied by contributing commercial banks. The data clearly shows an increase in both the time series volatility and the cross section dispersion of rates towards the end of the reserve maintenance period. These increases are highly correlated. With respect to quantities, we find that the volume of trade as well as the use of the standing facilities are also larger at the end of the maintenance period. These facts motivate the modelling strategy in the paper.

Most of the theoretical models of daily funds market equilibrium have evolved from the early seminal contribution by Poole (1968).<sup>1</sup> Some of the papers in this literature are Angeloni and Prati (1996), Bartolini, Bertola and Prati (2001, 2002), Henckel, Ize and Kovanen (1999), Pérez-Quirós and Rodríguez-Mendizábal (2003) and Woodford (2001). All these models share a main ingredient: the existence of a “liquidity shock” that creates uncertainty in the liquidity management of commercial banks and that we interpret in terms of imperfect information. Specifically, the idea is that commercial banks trade in the overnight funds market before they are able to determine their end-of-day balance with certainty. Furfine (2000) interprets this residual uncertainty as coming from “operational glitches, bookkeeping mistakes, or payments expected from a counterpart that fail to arrive before the closing of Fedwire”. In other words, credit institutions have less than perfect information and monitoring systems.

Usually these models are only concerned with the evolution of prices so they model representative agent economies. They are pure pricing models. However, in order to be able to explain the joint distribution of prices and quantities both in the time as well as in the cross section dimension, one needs to allow for *heterogeneity* plus some form of *market segmentation*. In this paper we provide a model with heterogeneous commercial banks subject to idiosyncratic shocks. These banks interact with the central bank through payment systems and the operational framework of monetary policy. The model provides a stylised representation of the relevant institutional features for the euro area. Commercial banks also interact with each other through the payments mechanism and the daily funds market. A form of market segmentation will prevent efficient netting, the verification of the law of one price, and will generate a distribution of interest rates across banks. It delivers testable propositions on

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<sup>1</sup> See also Baltensperger (1980) for a survey and references to the early literature on this subject.

the behaviour of interest rates over time and across banks, use of standing facilities and amounts traded. These are the propositions we confront with the empirical evidence.<sup>2</sup>

Market segmentation might look as an *ad hoc* proposition to model money markets, much more when banks exchange an extremely homogeneous good, reserves. However, talking with money market dealers of different private banks of the Euro area, it seems a plausible representation of the reality of money markets. The reasons usually argued by the practitioners not to have a single market are the existence of credit limits or agents playing a reputation game. According to the dealers, being short one day by a big amount is a piece of information they do not want to share with the general market. So, they prefer to pay more to settle their accounts privately with banks they usually do business with. Therefore, there are subgroups of trading banks that settle with each other before going to the general market or the standing facilities.

The design of the theoretical model is intended to replicate some of the basic features of the daily market for funds in the euro area. In doing so, we try to account for the most important elements of the operational framework for monetary policy. Our economy consists of a central bank and  $n$  commercial banks. These banks exchange overnight deposits in segmented markets and are subject to liquidity shocks. Commercial banks have to maintain a given level of required reserves on average during a reserve maintenance period. As in the Eurosystem, the central bank offers two standing facilities: a lending and a deposit facility. The two standing facilities define a corridor limiting the fluctuation in the overnight rate. We show that such an environment reproduces the main features of the market for funds in the euro area. In particular, the equilibrium in the model is characterized by rates whose time series as well as cross section volatilities increase towards the end of the maintenance period and are highly correlated. Furthermore, banks trade and use the standing facilities more at the end of the maintenance period.

The intuition behind these results is as follows. Every day banks face two risks. On the one hand, an unforeseen decrease in available funds may make a commercial bank use the lending facility. This is costly because the lending rate is larger than the market rate. On the other hand, in the opposite case of an unforeseen increase in available funds, a bank may fulfil reserve requirements early on during the reserve maintenance period, a case in which we say the bank is being “locked-in”. This is also costly because the marginal value of reserves accumulated beyond the requirement is the deposit rate which is smaller than the market rate. Thus, when banks determine how much funds to supply to the market

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<sup>2</sup> Furfine (1999), as Furfine (2000), uses transaction-level data. He looks at trading patterns and networks and finds evidence on the existence of relationship banking in the interbank market. Furfine does not explicitly address the theoretical modelling of bank heterogeneity together with market segmentation.

on every day of the maintenance period, they will look at how their decisions affect the probability of ending the day with an overdraft as well as the probability of being “locked-in”. Given an initial amount of reserves, if banks had the same supply of funds on every day, the probability of having an overdraft would be constant over time. However, the probability of having excess reserves would get larger as we approach the last days of the maintenance period. To compensate for this latter effect, banks try to hold relatively fewer reserves on the first days of the period and relatively more towards the end so as to reduce the probability of satisfying the reserve requirement early in the period.

The adjournment in the accumulation of reserves by banks has a further implication. As the end of the maintenance period gets nearer, the ability of banks to offset past shocks decreases. This makes banks more sensitive to shocks as time passes so that the elasticity of supply becomes a decreasing function of time. Thus, at the beginning of the period the supply of funds is very elastic. Banks are basically indifferent between different holdings of reserves, so they do not have the need to compensate liquidity shocks. They trade little and the interest rates are very similar both across banks and across time. On the last days of the period, though, supplies are inelastic so that shocks late in the period will have a larger effect on prices. Introducing market segmentation in this setting makes the cross section as well as the average across banks become more volatile towards the end of the reserve maintenance period. Furthermore, trade is also larger on the last days of the period as banks try to get their desired level of reserves. This larger trade happens both between banks and with the central bank through the use of the standing facilities.

The paper is structured as follows. In Section 2, we introduce the theoretical model. In Section 3 we solve the model and present the expressions that determine the demand for funds on each day of the maintenance period. Because the model cannot be solved explicitly, in Section 4 we perform numerical simulations in order to illustrate the properties of the model. Section 5 contains a description of the data set, which includes the daily contributions from individual banks used to compute the EONIA rate. The properties of the data are examined in Section 6. In Section 7 we model the time-series and cross-section behaviour of interest rates in the daily funds market in a systematic way. Finally, Section 8 concludes.

## **2. THE THEORETICAL MODEL**

In the model we consider  $n$  commercial banks and one monetary authority, the central bank. Commercial banks maintain deposits, called current accounts, with the central bank in order to fulfil reserve requirements and payments responsibilities. The operational framework of the central bank is composed of two elements. The first one refers to certain restrictions on the current accounts hold by commercial banks. Reserve balances of commercial banks cannot be negative by the end of each

trading session and the accumulated balance over each reserve maintenance period has to be large enough to meet required reserves, that is, it cannot be smaller than a number  $R > 0$ . This number  $R$  corresponds to the level of required reserves and is pre-determined. The reserve maintenance period has a length of  $T$  days. The second element consists of two standing facilities provided by the central bank. There is a lending facility where banks can borrow funds at the interest rate  $i^l$  and a deposit facility where banks can deposit funds at the rate  $i^d$ . These facilities are always available to commercial banks

The main decision each bank has to make on every day is to determine how much funds to trade on the money market. In deciding the amounts to borrow or lend in any session, banks take into account their reserve position. The position of bank  $j = \{1, 2, \dots, n\}$  is summarised by its current account with the central bank at the time it reaches the market on day  $t$ ,  $a_t^j$ , and the amount of reserves the bank has to accumulate from session  $t$  until session  $T$  to satisfy its reserve requirement, also known as its deficiency,  $d_t^j$ .<sup>3</sup> Assume that all banks start day 1 being identical, that is,  $d_1^j = D_1 = R$  and  $a_1^j = A_1$ . The pair  $s_t^j = (d_t^j, a_t^j)$  defines the individual state of a bank on any particular session.

In every session, banks are subject to shocks. In the general description of the model we distinguish between pre-trade shocks ( $\varepsilon_t^j$ ), that is, shocks that arise early in session  $t$ , before bank  $j$  exchanges funds in the market, and post-trade shocks ( $\lambda_t^j$ ), which represent late shocks that arise after trade takes place. We restrict these shocks to be i.i.d. across time. They follow the distributions:

$$\varepsilon_t^j \sim F_\varepsilon(\mu_\varepsilon^j, \sigma_\varepsilon^j)$$

and

$$\lambda_t^j \sim F_\lambda(\mu_\lambda^j, \sigma_\lambda^j),$$

where  $\mu_i^j$  is the mean of the shock  $i = \varepsilon, \lambda$  for bank  $j$  and  $\sigma_i^j$  is its standard deviation. Notice shocks are assumed to follow distributions that may differ across banks and not necessarily independent among them.

The timing of events between two sessions is as follows: consider a bank, say  $j$ , that is about to enter the market on day  $t$ . The bank has a deficiency  $d_t^j$  and an account balance of  $a_t^j$ . Let  $m_t^j$  and  $b_t^j$  be

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<sup>3</sup> In general, lower case letters refer to individual banks while capital letters refer to per capita market aggregates.

the amount of reserves kept by this bank and the ones loaned out to the market at session  $t$ , respectively. These magnitudes have to satisfy

$$m_t^j + b_t^j = a_t^j \quad (1)$$

and

$$m_t^j \geq 0. \quad (2)$$

After the bank leaves the market, it receives the late shock ( $\lambda_t^j$ ), so the balance at the end of the day in bank  $j$ 's current account is

$$r_t^j = m_t^j + \lambda_t^j.$$

The following day, the bank receives an early morning shock ( $\varepsilon_{t+1}^j$ ), so tomorrow's balance at the central bank by the time bank  $j$  goes again to the market is

$$a_{t+1}^j = a_t^j + \lambda_t^j + \varepsilon_{t+1}^j. \quad (3)$$

Notice that any reserves exchanged on any day have to be returned the following day. Additionally, we make the simplifying assumption that interest payments on borrowing and lending are not capitalised. Reserve deficiencies,  $d_t^j$ , evolve over time as

$$d_{t+1}^j = \max \{0, d_t^j - \max[0, r_t^j]\}, \quad (4)$$

for  $t < T$ . That is, the reserve balance of the bank at the end of the session ( $r_t^j$ ) is accumulated only if it is positive and deficiencies have a lower bound of zero. Because current accounts cannot be negative overnight, all reserves maintained in the central bank after the accumulated balance of a bank reaches the reserve requirement  $R$  are kept in excess and remunerated at the deposit rate.

Given market rates, the distribution of shocks, the distribution of banks over individual states and the evolution of those states, banks decide on borrowing and lending in order to maximise the expected sum of profits over all  $T$  sessions

$$E_1 \left( \sum_{t=1}^T \pi_t^j \right),$$

where  $E_1$  is the expectation with respect to the information set at the beginning of trading session 1. Let  $i_t^j$  be the interest rate at which bank  $j$  exchanges reserves in the market at the session  $t$ . The profits of this bank on any day, are

$$\pi_t^j = i_t^j b_t^j - c_t^j, \quad (5)$$

where  $i_t^j b_t^j$  are the revenues (costs) from lending (borrowing) funds in the market at the rate  $i_t^j$  and  $c_t^j$  represents the end-of-session costs derived from going to the deposit and lending facilities of the central bank.

The particular expression for the costs of using the facilities,  $c_t^j$ , depends on the day of the maintenance period. For the last day of the period, these costs are

$$c_T^j = i^l (d_T^j - m_T^j - \lambda_T^j) I[d_T^j \geq m_T^j + \lambda_T^j] - i^d (d_T^j - m_T^j - \lambda_T^j) I[d_T^j < m_T^j + \lambda_T^j], \quad (6)$$

where  $I[a]$  is an indicator function that takes value 1 when the statement in brackets is true. If the balance of the bank at the end of the day ( $m_T^j + \lambda_T^j$ ) is not enough to fulfil the deficiency (so that  $d_T^j \geq m_T^j + \lambda_T^j$ ), the bank will have to borrow the difference from the lending facility. Otherwise, if the bank accumulates excess reserves ( $d_T^j < m_T^j + \lambda_T^j$ ) they will be deposited at the deposit facility earning the rate  $i^d$ . Since in equilibrium the market rate is between  $i^d$  and  $i^l$ , the bank will use the facilities only if it has to. For the rest of days,  $t < T$ , the costs of using the facilities are

$$c_t^j = -i^l (m_t^j + \lambda_t^j) I[0 \geq m_t^j + \lambda_t^j] - i^d (m_t^j + \lambda_t^j - d_t^j) I[d_t^j < m_t^j + \lambda_t^j]. \quad (7)$$

If the balance of the bank at the end of the day is negative (that is,  $0 \geq m_t^j + \lambda_t^j$ ), the bank will have to borrow the necessary funds from the lending facilities to set it to zero. Otherwise, if the bank accumulates excess reserves ( $d_t^j < m_t^j + \lambda_t^j$ ) they will be deposited at the deposit facility earning the rate  $i^d$ .

### 3. SOLUTION OF THE MODEL

The way to solve this problem is by backward induction. We first solve for the individual supplies of funds in the market at time  $T$  and work backward towards the first period.



### 3.1 The last day of the reserve maintenance period ( $T$ )

The problem of bank  $j$  on the last day  $T$  can be written in dynamic programming form as follows. Let  $V_T(s_T^j; S_T)$  be the value function defined as the maximized profits of bank  $j$  at the last day of the period given its individual state  $s_T^j$  and the aggregate state  $S_T$ . This function satisfies:

$$V_T(s_T^j; S_T) = \max_{b_T^j} E_T(\pi_T^j) = \max_{b_T^j} E_T(i_T^j b_T^j - c_T^j), \quad (8)$$

with  $c_T^j$  defined in (6). The first order condition for a maximum is

$$i_T^j = i^l F_\lambda(d_T^j + b_T^j - a_T^j) + i^d [1 - F_\lambda(d_T^j + b_T^j - a_T^j)]. \quad (9)$$

This expression says that banks determine their supply of funds to the market in order to equate the marginal revenue of lending an additional unit of reserves with the expected marginal cost of that unit when reserves are needed, that is, when they are computed for the reserve requirement. These costs are  $i^l$  when the bank is overdrawn [with probability  $F_\lambda(d_T^j + b_T^j - a_T^j)$ ], and  $i^d$  when the bank has excess reserves [with probability  $1 - F_\lambda(d_T^j + b_T^j - a_T^j)$ ]. From this expression we observe that the supply of reserves of bank  $j$ ,  $b_T^j$ , is increasing with its initial level of reserves ( $a_T^j$ ), the rate at which the bank exchanges reserves in the market ( $i_T^j$ ), and it is decreasing with the reserve deficiency ( $d_T^j$ ), the lending rate ( $i^l$ ) and the deposit rate ( $i^d$ ). Furthermore, the partial derivative of the value function with respect to  $d_T^j$  is

$$\frac{\partial V_T(s_T^j; S_T)}{\partial d_T^j} = -i_T^j. \quad (10)$$

Remember from (9) that the supply of reserves on the last day of the maintenance period depends negatively on the level of deficiency. Therefore, the opportunity cost of starting the last day of the reserve maintenance period with one more unit of deficiency is equal to the market interest rate lost.

### 4.2 Days before the last ( $t < T$ )

Define the value function as

$$V_t(s_t^j; S_t) = \max_{b_t^j} E_t[\pi_t^j + V_{t+1}(s_{t+1}^j; S_{t+1})] = \max_{b_t^j} E_t[i_t^j b_t^j - c_t^j + V_{t+1}(s_{t+1}^j; S_{t+1})],$$

where the costs of using the facilities,  $c_t^j$ , are defined in (7). Using the evolution of  $d_{t+1}^j$  in (4), the first order condition with respect to  $b_t^j$  gives

$$\begin{aligned} i_t^j = i^l F_\lambda(b_t^j - a_t^j) + i^d [1 - F_\lambda(d_t^j + b_t^j - a_t^j)] - \\ - \int_{b_t^j - a_t^j}^{d_t^j + b_t^j - a_t^j} \left[ \int_{-\infty}^{\infty} \frac{\partial V_{t+1}(s_{t+1}^j; S_{t+1})}{\partial d_{t+1}^j} f_\varepsilon(\varepsilon_{t+1}^j) d\varepsilon_{t+1}^j \right] f_\lambda(\lambda_t^j) d\lambda_t^j. \end{aligned} \quad (11)$$

which generates the supply of funds on sessions  $t < T$ . Notice, it depends on the marginal value of reserves on the next session as measured by the partial derivative of the value function but only in those instances when the bank is accumulating reserves. As before, banks equate the marginal revenue with the marginal cost of supplying funds. For any  $t < T$ , the partial derivative of the value function with respect to  $d_t^j$  is

$$\begin{aligned} \frac{\partial V_t(s_t^j; S_t)}{\partial d_t^j} = -i_t^d [1 - F_\lambda(d_t^j + b_t^j - a_t^j)] + \\ + \int_{-\infty}^{b_t^j - a_t^j} \left[ \int_{-\infty}^{\infty} \frac{\partial V_{t+1}(s_{t+1}^j; S_{t+1})}{\partial d_{t+1}^j} f_\varepsilon(\varepsilon_{t+1}^j) d\varepsilon_{t+1}^j \right] f_\lambda(\lambda_t^j) d\lambda_t^j. \end{aligned} \quad (12)$$

The intuition of this expression is as follows. The cost of having a unit of deficiency more on day  $t$ , depends on the situation of the bank at the end of the day. If the bank ends up the day locked-in, starting day  $t$  with a larger deficiency has a low value, the deposit rate. This event happens with probability  $[1 - F_\lambda(d_t^j + b_t^j - a_t^j)]$ . This is the first term of the right hand side of (12). In all other cases, a larger deficiency increases  $d_{t+1}^j$  permanently and the value of this event is determined by  $\partial V_{t+1}^j / \partial d_{t+1}^j$ . This is the second term.<sup>4</sup>

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<sup>4</sup> It turns out that for days  $t$  far from  $T$ ,  $\partial V_t / \partial d_t \approx -i_t$ , so that from (11) the martingale hypothesis only holds conditioned on banks accumulating reserves. See Pérez Quirós and Rodríguez Mendizábal (2003) for an analysis of this point.

In general, explicit expressions for the solution of this model are not available for days  $t < T$ . In section 4 we present numerical simulations to show the main results. However, some intuition is available from the form of the expressions that define the solution. From (12) we see that in determining their supply of funds, banks care about the cases where the reserve problem is at a corner, that is, when the bank is overdraft ( $\lambda_t^j \leq b_t^j - a_t^j$ ) or “locked-in” ( $d_t^j + b_t^j - a_t^j \leq \lambda_t^j$ ). Because being “locked-in” is an absorbing state, the probability of reaching it increases over time which makes banks postpone the accumulation of reserves. Also, we can see that there will be use of the standing facilities before day  $T$ . This will happen if a bank has a negative current account at the end of the day (use of lending facility) or is locked-in (use of deposit facility). Finally, the distribution of interest rates and trade will depend upon the distribution of the individual states  $s_t^j = (d_t^j, a_t^j)$ . This means that changes in policy will, in general, affect the equilibrium distributions. In particular, level shifts in which the central bank changes the reserve requirement,  $R$ , in the same amount as the initial reserves,  $A_1$ , are not neutral. Changing the initial level of reserves together with reserve requirements affects the probability of reaching the corner states, and therefore, should affect the distribution of rates and quantities under any type of market structure.

#### 4. Simulations

We simulate an economy with a reserve maintenance periods of  $T = 3$  days and  $n = 12$  banks. Each of these banks are endowed with  $A_1 = 100$  units of reserves. The reserve requirement is  $R = 300$  for the whole period. This means that, in the aggregate, the system has enough reserves to satisfy the requirement. The lending rate is  $i^l = 5$  percent and the deposit rate is  $i^d = 3$  percent. This leaves a 200 basis point corridor as in the Eurosystem.

In order to generate a distribution of rates and quantities, the model will be solved for a variety of economies that differ upon the trade frictions we impose on market participants. The possibilities range from the frictionless case to autarchy. In the frictionless case banks meet in a single market place and all trades are centralised there. The equilibrium outcome of that economy can be characterised by a single interest rate and a distribution of quantities traded. This distribution depends on the heterogeneity of banks, that is, the levels of reserves they start the day with ( $a_t^j, j = 1, \dots, n$ ) and the deficiencies they have ( $d_t^j, j = 1, \dots, n$ ). As we will see below these two variables will determine the supply of funds for each bank and, therefore, the amounts transacted in the market. In autarchy there is no trade and a distribution of shadow interest rates is obtained. These shadow prices are defined as the marginal cost of borrowing or lending the first unit of reserves and, therefore, its distribution will also depend upon the heterogeneity of banks on each day.

Between these two economies we can generate additional ones in the following manner. Think of an economy where, because of search or transaction costs, markets are segmented. They are like islands not connected with one another. Each of these separated markets can in principle generate a different price and a different amount traded because their composition in terms of the agents involved may differ from one to another. So, if we look at the economy as a whole, it will be characterised by a joint distribution of prices and quantities. Next, we can perform the experiment of increasing the transaction/search costs. When these costs are zero or very small, the optimal size of the market is the whole economy. This is what we call the frictionless case. As we increase these costs the size of these markets decrease and the solution converges to the autarchic case. The Appendix provides the details about the computation of rates and quantities traded.

Before fully solving the model and to provide some intuition for the results below, we first compute the supply functions of an individual bank for a given sequence of expected market rates. Figure 1 presents the supply functions of the “average” bank in the frictionless economy. For each day, it is assumed that the bank has not received any shock on previous days. In order to compute these functions, we have to provide the expected rate in the market. To keep the functions comparable between days, we assume that the expected rate is equal to the middle of the band  $(i^l + i^d)/2$ . We see how the elasticities are reduced gradually over time. The shape of these functions suggests that at the beginning of the maintenance period, banks are indifferent with respect to their reserve level. This means that they will not try to compensate liquidity shocks and the interest rate will be stable both over time and across banks. As the end of the period gets nearer, though, demands become more inelastic so banks have clearer targets with respect to a particular level of reserves. This implies more trade and more volatility of rates.

To solve for the equilibrium prices and quantities we carry out a Monte Carlo experiment where we simulate  $Z = 1000$  reserve maintenance periods identical to the one described at the beginning of the section. With respect to the shocks, we consider late shocks  $(\lambda_t^j)$  only. Here,  $\lambda_t^j$  represents the uncertainty about the changes in the current account by the end of the day because of transfers that take place later in the day, clerical errors and the like. To construct these shocks, define as  $\lambda_t^j(k)$  the random transfer of funds between banks  $k$  and  $j$  or the errors committed in accounting for a known transaction between these two banks. This transfer is assumed to be normal with mean 0 and standard deviation  $\sigma$ . If the shock is positive, funds are moved from bank  $k$  to bank  $j$  and if negative, bank  $j$  moves reserves to bank  $k$ . Assume  $\lambda_t^j(j) = 0$ . The shock  $\lambda_t^j$ , is, therefore equal to

$$\lambda_t^j = \sum_{k=1}^n \lambda_t^j(k)$$

which follows a normal distribution with zero mean and standard deviation

$$\sigma_j = \sigma \sqrt{n-1}.$$

It is easy to verify that these shocks always sum up to 0 across banks, so no reserves enter or leave the system in the aggregate and have covariances and correlations equal to

$$E_t(\lambda_t^j \lambda_t^k) = -E_t[(\lambda_t^j(k))^2] = -\sigma^2$$

and

$$\rho_t(\lambda_t^j, \lambda_t^k) = \frac{E_t(\lambda_t^j \lambda_t^k)}{\sigma_j \sigma_k} = \frac{-1}{n-1},$$

respectively. In our simulation, the standard deviation of individual transfers is  $\sigma = 20$  which implies that banks are subject to shocks which are jointly normal with standard deviation of  $\sigma_j = \sigma (n-1)^{1/2} = 66.33$  and a correlation between shocks of  $\rho_t(\lambda_t^i, \lambda_t^k) = -0.09$ .

The rationale to model shocks in this way is as follows. Even if no reserves enter or leave the system, there is aggregate uncertainty in this economy. The reason is that, given the finite number of banks, prices and trade are not going to be independent on how reserves are reshuffled among them. In this sense, market segmentation will play a crucial role in linking different distributions of reserves into different prices. We will show that even in this simple case with no shocks that change the overall supply of reserves, the statistical properties of prices and quantities traded follow the stylised facts to be found from the data.

Tables 1, 2 and 3 report descriptive statistics for the state of the system defined by the initial levels of reserves  $(a_t^j)$ , average daily deficiencies  $[d_t^j / (T-t+1)]$  and trade  $(b_t^j)$  for sessions 2 and 3 and for different sizes of the groups  $h = \{2, 3, 4, 6, 12\}$ . Let  $d_t^j(z)$  be the average daily deficiency of bank  $j = \{1, \dots, n\}$  in session  $t = \{1, \dots, T\}$  of the simulation  $z = \{1, \dots, Z\}$ . The column labelled “mean” in Table 2 corresponds to the average deficiency and is computed as

$$mean(d_t^j) = \frac{1}{Z} \sum_{z=1}^Z \left[ \frac{1}{n} \sum_{j=1}^n d_t^j(z) \right].$$

The column labelled “ $\sigma_{ts}$ ” is the standard deviation of the average deficiency computed across simulations, that is,

$$\sigma_{ts}(d_t^j) = \sqrt{\frac{1}{Z} \sum_{z=1}^Z \left( \frac{1}{n} \sum_{j=1}^n d_t^j(z) - mean(d_t^j) \right)^2}.$$

Finally, the column labelled “ $\sigma_{cs}$ ” is the average of the cross section standard deviation, that is,

$$\sigma_{cs}(d_t^j) = \frac{1}{Z} \sum_{z=1}^Z \sqrt{\frac{1}{n} \sum_{j=1}^n \left( d_t^j(z) - \frac{1}{n} \sum_{j=1}^n d_t^j(z) \right)^2}.$$

Computations for  $a_t^j$  and  $b_t^j$  in Tables 1 and 3 are equivalent.

The initial level of reserves ( $a_t^j$ ) is an exogenous variable. This means that its distribution does not depend on the market structure as it is shown in Table 1. Because all banks start identical, deficiencies on  $t = 2$  are also exogenous, though. On  $t = 3$  as market groups get larger, deficiencies become smaller and less volatile as banks take advantage of trade. We observe that, for a given size of the group, the volatility of aggregate variables like the equilibrium interest rate or aggregate trade is larger on the last day. Also, we see that as the size of the group increases, the aggregate uncertainty decreases. Table 3 shows that banks respond to the larger uncertainty with more trade (compare columns for  $t = 2$  and  $t = 3$  in Table 3). As the size of the groups increase there is also more trade, which is why aggregate uncertainty decreases for the economy as a whole.

Table 4 reports statistics for the interest rate. First, we compute a measure of the aggregate interest rate for this economy. For that, we select banks that make positive loans and compute a weighted average of their rates. The weights are the percentage of the loans over the whole amount transacted that session.<sup>5</sup> We compute the square of the change between the aggregate interest rate that session and the value on the previous one. The average of that series shows in the column labelled  $\sigma_{ts}$ . It gives an indication of how volatile the aggregate interest rate is in the time series dimension. We also compute

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<sup>5</sup> This is the way the EONIA is computed.

the cross section volatility of rates for banks that make loans in the market. The average of the corresponding cross section variance is reported in the column  $\sigma_{cs}$ . It gives an indication on how disperse rates are. We observe, the aggregate interest rate is more volatile on the last day, both in the time as well as in the cross section dimension. Furthermore, as groups become larger, the volatility decreases, since there is more room for trade.

In Table 4 we also show the correlation between both measures of dispersion in the column  $\rho(ts,cs)$ . We report two ways of calculating this correlation. The first one is constructing a time series of measures of dispersion and look at their correlation. The result is collected in the column  $t = 2, 3$ . The second one is for each session separately. Both measures of dispersion are very positively correlated as we find in the data.

Tables 5 and 6 present the use of the standing facilities for different days and size of market groups. Table 5 shows the probability of going to the lending and deposit facilities. For each realisation for the Monte Carlo simulation, this probability is computed for each bank given the distribution of shocks and then those probabilities are averaged over banks and simulations. Table 6 includes the expected use that all banks will make of the facilities, again averaged over all realisations of the simulation. As mentioned before, on the first day there is no trading so the use of the standing facilities is exogenous and, therefore, independent of the size of the market groups. We see that the recourse to the standing facilities is both more likely and larger on the last day of the maintenance period than on previous days. It is interesting to notice that the use of the lending facility is very similar for the three days, while the use of the deposit facility peaks on the last day as we observe in the data. Finally, as market groups become larger, banks find more opportunities to trade among themselves and go less to the standing facilities.

Out of the theoretical model and the simulation results we can draw a set of conclusions relevant for the empirical analysis. It is a clear prediction of the model that interest rate volatility increases towards the end of the maintenance period. The increase in volatility in a time series dimension with the evolution of the maintenance period is not new in the literature. A large number of papers analyse this phenomenon. Hamilton (1996), Wurtz (2001) are just two examples for the US and the European case respectively. However, the cross-section definition of interest rates, that is, departures from the law of one price, have not been considered in the literature. The omission may have been due to lack of relevant data. Associated with these larger price volatilities we find a larger volume of trade, both in the market across banks and with the central bank through the use of the standing facilities. This larger trade is also concentrated at the end of the reserve maintenance period.

## 5. DESCRIPTION OF THE DATA

The data used in this study consists of interest rates obtained by the major European banks when they lend funds in the overnight market. In particular, each data point represents the average interest rate charged in that day by each lending bank. The sample covers 64 banks from January 4<sup>th</sup> 1999 to November 9<sup>th</sup> 2002. This data set was provided by the European Banking Federation (EBF) and is the one used to compute the time series for the EONIA. The number of observations on a particular day may be smaller than the number of banks, though, simply because some banks may not be active on that day or because they are borrowing. Furthermore, the number of banks in the panel is not always the same because the EONIA panel has changed in different occasions to include or exclude some banks.

Most of the papers analysing the market microstructure (see Hartmann, Manzanares and Manna (2000) as an example) usually construct their databases from quotes of the brokers in Reuters. This has several problems. First, they only cover the part of the market that is traded through brokers, missing the larger transactions that are usually done directly between banks. Additionally, brokers, when the market is very active, do not necessarily report the quotes to Reuters. Therefore, another big part of the market is missing. Finally, the quotes do not oblige the counterparts to trade, much more when the overnight market is uncollateralized, which implies that different rates are charged to different banks because of the different risks associated with particular transactions. On the contrary, in the EBF database the observations correspond to actual trades.

Obviously there are some drawbacks to the EBF database. First, we only have information for average rates during the day. That characteristic makes it not possible to say anything about the intra-day activity. Second, we have only information about the lenders, not the borrowers. We cannot test any implication that theory could tell us about the borrowers' behaviour.

## 6. SOME PROPERTIES OF THE DATA

Table 4 collects most of the testable implications of the theoretical model. We observe positive correlation of cross-section and time series volatility for the whole sample whatever is the group size (column 1, labelled  $\rho(ts,cs)$  for  $t=2,3$ ). In addition, in any given day, we also observe this positive correlation, for most of the group sizes, particularly striking at the end of the reserve maintenance period (columns 4 and 7, labelled  $\rho(ts,cs)$  for  $t=2$  and for  $t=3$  respectively). The implications of these positive correlations on the data are stronger than the associated to the whole sample (column 1). As shown in Table 4, the theoretical model implies that the positive correlation is associated, not only to the fact that both variances increases at the end of the maintenance period, but also, given that for any



given day we observe positive correlation of variances, that days with higher (lower) time series volatility than the expected for that day, will come associated with higher (lower) than expected cross-section volatility. Therefore, the strongest consequence of the model is the positive correlation of the shocks to both volatilities.

However, before testing the implications of the model, we should discard some alternative explanations that could imply the same predicated properties. For example, one of the key assumptions of the model is the fact that no bank dominates the market. If a set of banks dominate the market we could think on strategic behaviour that banks could play to increase rates when the system is short, affecting the volatility of rates. On the contrary, the model assumes that banks are equal at the beginning of the maintenance period and they differ from each other in the path of the shocks that they receive over the maintenance period. If this is the case, individual excess profits, calculated as the interest rate obtained by the lending of their reserves minus the average rate obtained for those reserves, should be uncorrelated across maintenance periods. We test this hypothesis and we accept it at 5 percent in all but two of the banks of the sample. However, the models allows (and actually predicts) that profits for a given bank, maybe correlated over time, within a reserve maintenance period. We test this hypothesis and we reject the null of no correlation in more than 20 percent of the banks. We conclude from these findings that even though banks play different roles in different maintenance periods, idiosyncratic shocks allow them to keep advantage positions within the reserve maintenance period.

Another way of testing the non-strategic behaviour of banks, as assumed by the model, is to decompose the excess profits<sup>6</sup> obtained by a bank in a given maintenance period between the profits obtained by “market power” (i.e. on a given day, how much a bank makes for its reserves in excess of the market rate) and the “timing” (i.e. on a given maintenance period, how much a bank makes because it lends when the rates are higher than the average rate of the period). If the assumptions of the model are correct, these two sources of excess profits should be uncorrelated, because no bank should have a strategic behaviour in playing the markets game. These assumptions are corroborated by the data. No bank seems to be systematically playing the game of waiting until the rates are high to get advantage of its market power and the correlation between these two sources of excess profits is even negative (-0.13).

An additional assumption of the model is that a three period maintenance period is a good representation of the one-month maintenance period. If three periods are enough, there should not be

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<sup>6</sup> Excess profits is defined as the returns obtained by a bank in a given maintenance period and the average return obtained by the funds on that period.

any differences in the density function of the data in the first  $T-2$  periods of the maintenance period, (where  $T$  is the number of days of the maintenance period). The idea is that those  $T-2$  periods can be considered repetitions of the same distribution. In order to address this question empirically, we need to analyse the density function of the rates on different days of the maintenance period. We have information on the different rates that banks have obtained for their reserves and we estimate the distribution function for each day of the maintenance period. We consider as the same day days that have the same distance to the end of the reserve maintenance period. For example, to calculate the distribution function of the last day of the reserve maintenance period, we use all the 45 distributions of the end of the maintenance period days that we have in the sample. We do the same for all the other days. At the end we have 20 distributions; for the last day of the maintenance period, the day before, up to the day  $t-19$ . Given that, on average we have around 40 banks active on a given day, this implies an average of 1800 observations for each of these distributions. We calculate the Kolmogorov-Smirnov (KS) test of the equality of these 20 distributions. We accept the null that distributions are the same (doing pairs of KS tests) for all days of the maintenance period but the last 4. A good way of checking the intuition behind this result can be seen in Figure 2. In this Figure we plot in a two-dimensional scale the distance between the KS statistics for each pair of distributions. Obviously, these distances cannot be represented in an Euclidean space because they are 20-dimensional. However, we use multi-dimensional scaling to reduce these dimensions to two. The purpose of this technique is to obtain the two-dimensional projection of a  $N$  dimensional space. The intuition is that the Euclidian distances measured in the plane will be an approximation of the distances in the  $N$ -dimensional space. Points which are close in the plot will be close in the actual measure of the distance across the distributions. As we can see from the plot, the last days are away from the group of the rest of the days. Empirical evidence seems to corroborate that just using a few days is not a bad approximation of what is going on in the markets.

Finally, an important question needs to be addressed: Does the “law of one price” hold in this market? This is a key question, because all the theoretical analysis tries to explain deviations from this law. The data seems to confirm the relevance of the problem addressed by the theory. Figure 3 plots different measures of spreads across banks, standard deviations of rates (weighted by the value transacted and not weighted) on a given day and difference between rates corresponding to the ninth and the first deciles. All these measures contain the same message. Even though the asset transacted in the market is extremely homogeneous (reserves) different prices are paid at different moments for this homogeneous good. There is a clear violation of the law of one price.

One important caveat should be recognised. In our data, it may appear as if the law of one price is violated if rates vary during the day and banks lend funds at different hours. Given that banks reports only daily averages can this be the source of the dispersion of interest rates in the dataset? In order to

address this question, we need to depart from our database and check for other sources of information on daily patterns. We are reluctant to use Reuters data because we know that they are not binding and that most of the transactions, when there is more action on the market, are made by direct contact across banks. We prefer to use Italian electronic market data (emid). This database captures quotes that are binding for the banks, giving us some more accurate information on the actual situation of the market. We have tick data for six months (from March to September 2002) which represent around 260,000 observations. In order to check the dispersion of the markets in each period of time we calculate volatilities of quotes every five minutes. These volatilities are plotted in Figure 4. As we can see, the volatility of rates is an end-of-the-day phenomenon. Figure 5 shows the average variation of the average rate quoted every five minutes with respect to the previous five minutes and the previous hour. Looking at this graph, it is clear that the variation of rates occurs at the end of the day. We conclude that dispersion across banks cannot be due to the fact that banks go to the market at different moments. The law of one price holds for most of the day and it seems to be at the end of the day when we observe the dispersion of rates across banks. We can then claim that different prices across banks cannot be explained by any intra day pattern.

Based on the previous analysis, it seems that the assumptions of the theoretical model are supported by the data. The fact that we have presented evidence against a big set of alternative explanations for the pattern of the rate allows us to concentrate on searching for empirical support to the conclusions of the model. These conclusions are summarized by the increase in volatility of rates both in the time series as well as in the cross section dimension as we get closer to the end of the maintenance period together with the positive correlation between both volatilities. Additionally, Furthermore, there should be an increase in trade in the market and a larger use of both facilities also at the end of the reserve maintenance period.

## **7. TIME SERIES AND CROSS SECTION VOLATILITY**

The main goal of our analysis is to study the relation between time series and cross section volatility of rates. We measure time series volatility as the absolute variation of rates from a day to the next. For the cross section volatility we use the standard deviation of rates where each of the rates obtained by a bank is weighted by the amount transacted on the day.

The first prediction of the theory is the fact that volatility increases at the end of the maintenance period both in a time series and in a cross section dimension. This fact can be clearly appreciated in Figure 6, where we represent the average absolute variation of rates in different days of the maintenance period and in Figure 7, where we represent the cross section volatility.

One way to see how close these two volatilities move together would be to calculate the correlation between the actual time series volatility (variation in overnight rates squared) and the definition of the cross section volatility that we consider (in our case, the standard deviation of the rates on a given day). The correlation between these two series is indeed high, namely 0.62. However, a few caveats should be taken into account when interpreting this number. Several features of the market may affect both volatilities in the same direction. From the theoretical model we have learned that some explanatory variables, particularly end of reserve maintenance periods should affect the time series and cross section volatility in the same direction. Similarly, other calendar effects associated with larger than average volatilities of payments like end of the months, end of the year, etc... may also be important. However, from the theoretical model, we have learned that the correlation between these two volatilities should be deeper than the one that comes from the fact that some exogeneous variables, as the calendar effect, affect both volatilities in the same direction. We expect that unexpected movements in the volatility of one of the variables will be correlated with unexpected movements in the same direction of the other volatility, as stated in table 4. Looking at the different tables of the simulation exercise, we have learned that, not only cross section and time series move together in relation with time in the maintenance period, but shocks to the liquidity of the system shift away the time path of both series at the same time. According to theory, the correlation should be positive and strongly significant. This conclusion could represent the deepest consequence of our theory, the fact that not only expected volatilities move together but shocks to these volatilities also move in the same direction.

In order to separate the expected of the unexpected component of the volatility of each of the variables we estimate a reduced form model where we include the explanatory variables described in the theoretical part.

The estimated model is the following:

$$i_t = i_{t-1} + \beta' X_t + h_t \varepsilon_t \quad (13)$$

$$\ln(h_t) = \lambda V_t + \xi u_t + \sum_{j=1}^n \left[ \delta_{j,1} (\ln(h_{t-j}) - \lambda V_{t-j}) + \delta_{j,2} \frac{\varepsilon_{t-j}}{\sqrt{h_{t-j}}} + \delta_{j,3} \left( \frac{\varepsilon_{t-j}}{\sqrt{h_{t-j}}} - \sqrt{\frac{2}{\pi}} \right) \right] \quad (14)$$

$$\ln(\text{var}_t) = \lambda' V_t + \sum_{j=1}^n [\delta_{j,1} (\ln(\text{var}_{t-j}) - \lambda' V_{t-j})] + u_t \quad (15)$$

$$\varepsilon_t \sim pN(0,1) + (1-p)N(0,\sigma_1) \quad u_t \sim N(0,\sigma_2) \quad (16)$$

where the correlation of  $\varepsilon_t$  and  $u_t$  is equal to 0. The variables  $X_t$  and  $V_t$  represent the set of calendar dummies that affect the mean and the volatility of the rates. These variables are carefully defined in Table 7. This model follows Pérez Quirós and Rodríguez Mendizábal (2003) but has been extended to consider the cross section dimension of our problem. The mixture of normals for the time series volatility equation is standard in the literature of the empirical analysis of the overnight rate to capture the long tails of the distributions of rates. We find that this mixture is not necessary when describing the cross section volatility of rates.

It is important to point out that an estimation like the proposed one has no implications on the direction of causality from the time series to the cross section volatility or viceversa. It would be equivalent to estimate the previous equations allowing for the shocks to be correlated. The estimated coefficient  $\xi$  will be the estimated correlation of the unexpected shocks to the variances. It turns out it is positive and extremely significant (p-value = 0.00).

The results of estimating jointly (13), (14), (15), and (16) are displayed in Table 7. According to our statistical analysis the main implications of the theory are supported by the data. Cross section and time series volatility are highly correlated even controlling for all the explanatory variables that explain the joint behaviour of both. One more implication can also be added. As predicted by Tables 5 and 6, the use of the facilities is also closely related to the end of the maintenance period. Figure 8 plots the average use of these facilities on different days of the maintenance period. The graph in this case is probably more illustrative than any formal test that we might do; as predicted by the theory, the use of these facilities is clearly linked to the end of the maintenance period, with a stronger link in the case of the deposit facility.

## 8. CONCLUSIONS

In this paper we provide a parsimonious model of the daily funds market equilibrium. We build on the original seminal contribution made by Poole (1968). The essential feature of the models of overnight market equilibrium, in the tradition of Poole, is that commercial banks do not know their end-of-day position at the time of trading. This reflects the fact that credit institutions have imperfect information and monitoring systems. In addition, we assume market segmentation and banks subject to heterogeneous liquidity shocks banks to generate a distribution of prices and quantities traded across banks. With these assumptions, the main prediction of the model is a high positive correlation between the conditional time series volatility and the cross section dispersion of interest rates. The reason for this result resides in the reduction of the elasticity of the supply of funds by banks as the system approaches the end of the reserve maintenance period.

In our research we used the EONIA panel database, kindly made available by the European Banking Federation (EBF). The database includes daily information on the lending rates applied by contributing commercial banks. Interest rates correspond to actual trading rates. After ruling out alternative explanations, the data clearly confirms the prediction of the theory: There exists a strong positive correlation between the average time series volatility and the cross section dispersion of interest rates for the whole sample and also conditional to the day of the maintenance period. Therefore, periods with higher (lower) than expected time series price volatility come associated to periods with higher (lower) than expected cross section dispersion of rates.

One immediate implication of the model concerns the level of reserves and the timing of liquidity provision by the central bank. With the nonlinearities imposed by the operational framework of monetary policy when and how much liquidity is made available become relevant. In particular, a change in reserve requirements compensated by an increase in liquidity is not neutral and should affect the distribution of rates and quantities traded in the market. Additionally, the model is fully flexible on many dimensions and could be extended to analyse other issues in this market. For example, we could distinguish between large and small banks, and between universal and investment banks. It could also consider networks of banks structured in a systematic way. It is a promising area for further work.

## APPENDIX

This appendix explains the determination of the market rate and the quantities traded in each of the market settings considered in the paper.

**Case a: Frictionless economy.** The particular form of the equilibrium solution depends upon the market structure considered. First, assume that all trades are centralized in a market place and there is perfect information about banks situation in terms of the values of  $s_T^j$  and its distribution. In this case, there is a single interest rate that clears the market,  $i_T^j = i_T$ . From (9) the supply of funds by bank  $j$  on the last day of the reserve maintenance period is

$$b_T^j = a_T^j - d_T^j + F_\lambda^{-1} \left( \frac{i_T - i^d}{i^l - i^d} \right)$$

where  $F_\lambda^{-1}$  is the inverse of the distribution function  $F_\lambda$ . In the market, loans are in zero net supply so, aggregating supplies

$$0 \equiv \sum_{j=1}^n b_T^j = \sum_{j=1}^n a_T^j - \sum_{j=1}^n d_T^j + n F_\lambda^{-1} \left( \frac{i_T - i^d}{i^l - i^d} \right),$$

which yields the equilibrium rate

$$i_T = i^d + (i^l - i^d)F_\lambda(D_T - A_T). \quad (A1)$$

In this expression  $D_T$  and  $A_T$  are the per capita level of deficiency and initial reserves, respectively:

$$D_T \equiv \frac{1}{n} \sum_{j=1}^n d_T^j; A_T \equiv \frac{1}{n} \sum_{j=1}^n a_T^j.$$

Then, the supply of funds by bank  $j$ ,  $b_T^j$ , becomes

$$b_T^j = (D_T - A_T) - (d_T^j - a_T^j), \quad (A2)$$

so a bank will be supplying or demanding reserves in the market depending on whether the “individual excess reserves”  $d_T^j - a_T^j$  are below or above the aggregate value. Notice a bank can have excess reserves,  $d_T^j < a_T^j$ , and still borrow them because the average value is even lower. As we have seen, in that situation reserves will be cheap.

Although prices and quantities can be computed exactly on the last day of the maintenance period, it is not so on other days. For  $t < T$ , solution is approximated using a grid for the individual shocks. Then an iteration procedure is used together with (11) to find the sequence of market rates for those days.

**Case b: Autarchy.** Now assume that banks are on their own so no trades are possible. Since banks are isolated, for the last day of the maintenance period  $b_T^j = 0$  and the shadow rate is

$$i_T^j = i^d + (i^l - i^d)F_\lambda(d_T^j - a_T^j). \quad (A3)$$

For the previous days  $b_t^j = 0$  and (11) is used together with the grid of the shocks to approximate the distribution of rates.

**Case c: Market groups.** In order to generate intermediate cases, where there is a cross section of both, prices and quantities, assume we form random groups of size  $h$ . If  $h$  is  $n$  we are in the frictionless market (case a). If  $h$  is 1 we have autarchy (case b). Since each group is isolated in terms of trading of

funds we take each of them as a single economy and apply the formulas for the frictionless case. However, because the shocks move reserves among the groups, the aggregate state of each group may change on each day. The Montecarlo simulation is just a long series of possible trajectories for these initial states.

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**Table 1**Descriptive statistics for initial reserves ( $a_t$ )

	$t = 2$			$t = 3$		
$s$	mean	$\sigma_{ts}$	$\sigma_{cs}$	mean	$\sigma_{ts}$	$\sigma_{cs}$
2	100	-	64.37	100	-	91.56
3	100	-	64.37	100	-	91.56
4	100	-	64.37	100	-	91.56
6	100	-	64.37	100	-	91.56
12	100	-	64.37	100	-	91.56

**Table 2**Descriptive statistics for daily deficiencies ( $d_t$ )

	$t = 2$			$t = 3$		
$s$	mean	$\sigma_{ts}$	$\sigma_{cs}$	mean	$\sigma_{ts}$	$\sigma_{cs}$
2	104.3	5.08	33.80	114.7	13.11	89.69
3	104.3	5.08	33.80	111.6	10.20	81.29
4	104.3	5.08	33.80	110.3	8.77	77.01
6	104.3	5.08	33.80	109.2	7.51	71.69
12	104.3	5.08	33.80	108.1	6.21	66.48

**Table 3**Descriptive statistics for lending ( $b_t$ )

	$t = 2$			$t = 3$		
$s$	mean	$\sigma_{ts}$	$\sigma_{cs}$	mean	$\sigma_{ts}$	$\sigma_{cs}$
2	296.5	60.91	77.30	492.5	37.47	101.6
3	381.3	74.18	92.06	584.7	44.13	121.8
4	414.4	77.87	95.56	635.3	47.59	131.0
6	441.7	82.33	100.7	687.4	50.42	140.1
12	464.1	86.61	106.4	749.4	57.62	155.6

**Table 4**

Descriptive statistics for interest rates

	$t = 2, 3$	$t = 2$			$t = 3$		
$s$	$\rho(ts,cs)$	$\sigma_{ts}$	$\sigma_{cs}$	$P(ts,cs)$	$\sigma_{ts}$	$\sigma_{cs}$	$\rho(ts,cs)$
2	0.1393	0.0705	0.3337	-0.0048	0.1021	0.5834	0.1417
3	0.2170	0.0385	0.3177	0.2111	0.0573	0.4818	0.1754
4	0.3293	0.0267	0.2832	0.3079	0.0434	0.4138	0.3065
6	0.3247	0.0145	0.2078	0.4496	0.0272	0.2831	0.2812
12	-	0.0047	-	-	0.0028	-	-

**Table 5**

Average probability of using the standing facilities

	Lending			Deposit		
$s$	$t = 1$	$t = 2$	$t = 3$	$t = 1$	$t = 2$	$t = 3$
2	0.0658	0.1157	0.1524	0.0013	0.1697	0.4807
3	0.0658	0.1090	0.1405	0.0013	0.1380	0.4788
4	0.0658	0.1038	0.1376	0.0013	0.1204	0.4773
6	0.0658	0.0990	0.1369	0.0013	0.0993	0.4705
12	0.0658	0.0918	0.1428	0.0013	0.0745	0.4564

**Table 6**

Average expected use of the standing facilities

	Lending			Deposit		
$s$	$t = 1$	$t = 2$	$t = 3$	$t = 1$	$t = 2$	$t = 3$
2	29.02	30.74	32.01	18.71	33.92	72.85
3	29.02	30.45	31.39	18.71	31.57	65.85
4	29.02	30.25	31.32	18.71	30.50	61.53
6	29.02	30.08	31.34	18.71	29.54	56.52
12	29.02	29.84	31.65	18.71	28.53	50.43

**Table 7**

Estimation of equations (13), (14), (15) and (16)

A: Mean parameters				
	Time series		Cross section	
	Estimation	Std. Error	Estimation	Std. Error
$\beta_{1t}$	-0.213	(0.049)	-	-
$\beta_{2t}$	-0.679	(0.763)	-	-
$\beta_{3t}$	4.296	(0.493)	-	-

B: Variance parameters				
	Time series		Cross section	
	Estimation	Std. Error	Estimation	Std. Error
$\lambda_{1t}$	-6.481	(0.746)	-9.810	(0.061)
$\lambda_{2t}$	3.040	(0.273)	2.491	(0.113)
$\lambda_{3t}$	0.589	(0.267)	-	-
$\lambda_{4t}$	1.873	(0.412)	2.606	(0.683)
$\lambda_{5t}$	1.206	(0.296)	1.456	(0.149)
$\lambda_{6t}$	0.165	(0.200)	0.113	(0.076)
$\lambda_{7t}$	0.237	(0.248)	0.428	(0.110)
$\lambda_{8t}$	0.833	(0.267)	-	-
$\xi$	0.403	(0.045)	-	-
$\delta_{11}$	0.209	(0.039)	0.359	(0.022)
$\delta_{12}$	0.541	(0.125)	0.049	(0.023)
$\delta_{13}$	1.079	(0.227)	-	-
$\delta_{21}$	0.030	(0.042)	-	-
$\delta_{21}$	0.217	(0.051)	-	-
$\sigma$	0.556	(0.043)	1.476	(0.149)
$p$	0.036	(0.004)	-	-

The parameters  $\beta_{jt}$ ,  $j = 1, 2, 3$ , are the coefficients of the variables  $X_{jt}$  in equation (13) while the parameters  $\lambda_{jt}$ ,  $j = 1, 2, \dots, 9$ , are the coefficients of the variables  $V_{jt}$  in equation (14) where:

$X_{1t}$ = Constant	$X_{2t}$ = End of maintenance period	$X_{3t}$ = Beginning of maintenance period
$V_{1t}$ = Constant	$V_{2t}$ = End of maintenance period	$V_{3t}$ = Beginning of maintenance period
$V_{4t}$ = End of year	$V_{5t}$ = End of month	$V_{6t}$ = Friday
$V_{7t}$ = Council meeting	$V_{8t}$ = Day after council meeting	

The parameters  $\delta_{11}$  to  $\delta_{21}$  also appear in equation (14). The variable “End of maintenance period” includes the four last days of the MP, except when the MP ends on a Wednesday. In this case, there is no end of maintenance period effect because the banks have the last open market operation to adjust their balances. Similar estimation results are obtained when the variable “End of maintenance period” only includes the days after the last open market operation.

FIGURE 1  
Excess supply functions for average bank in the nonlinear case

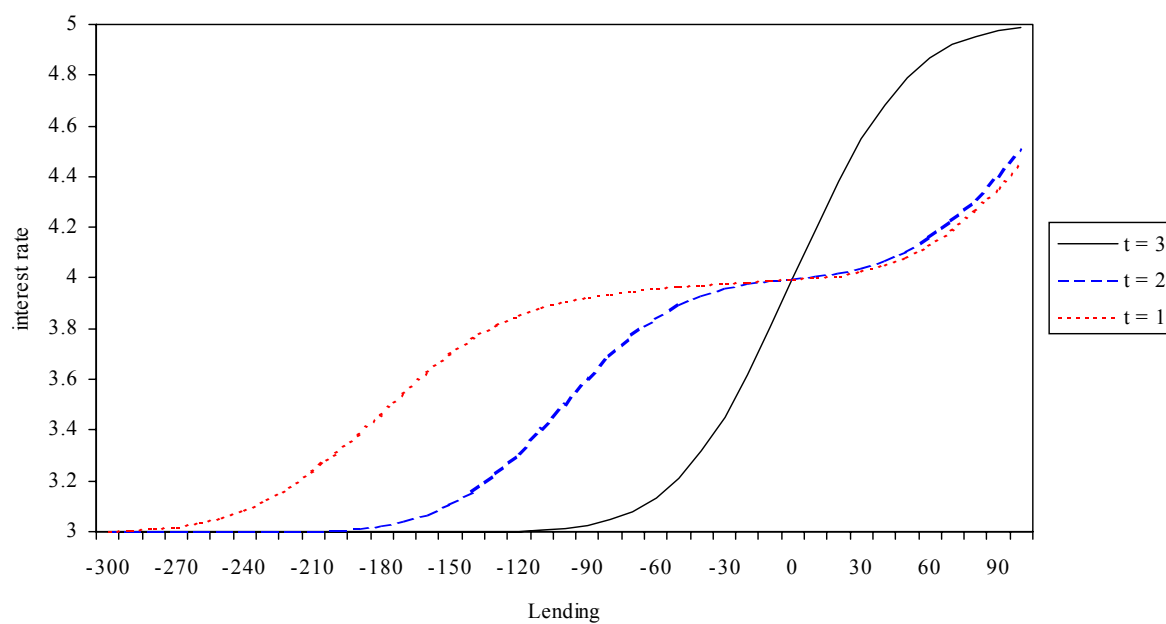
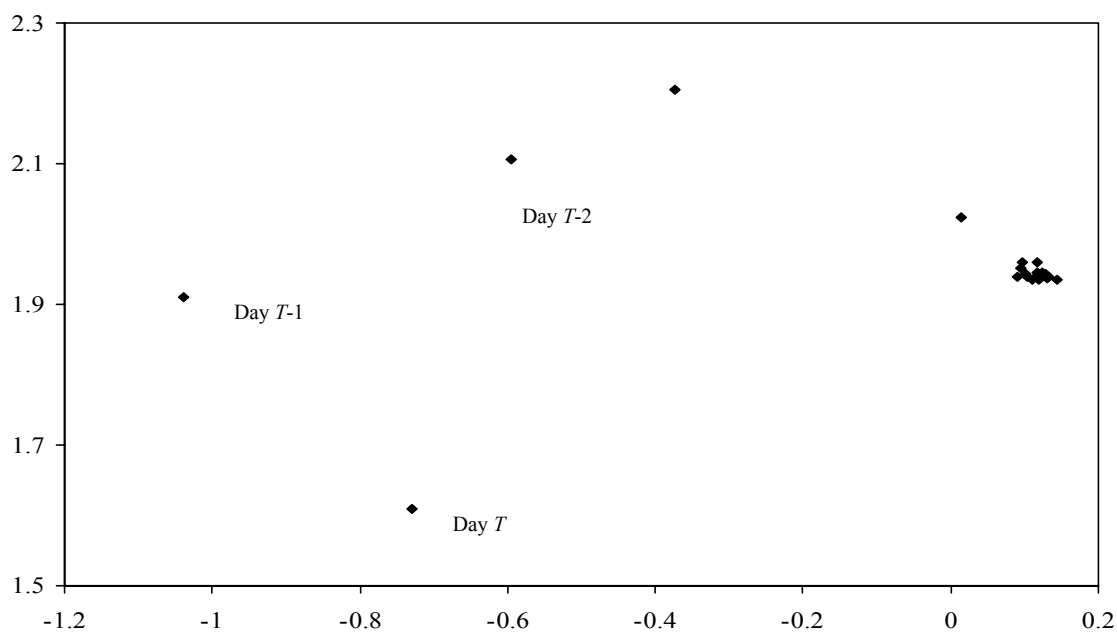


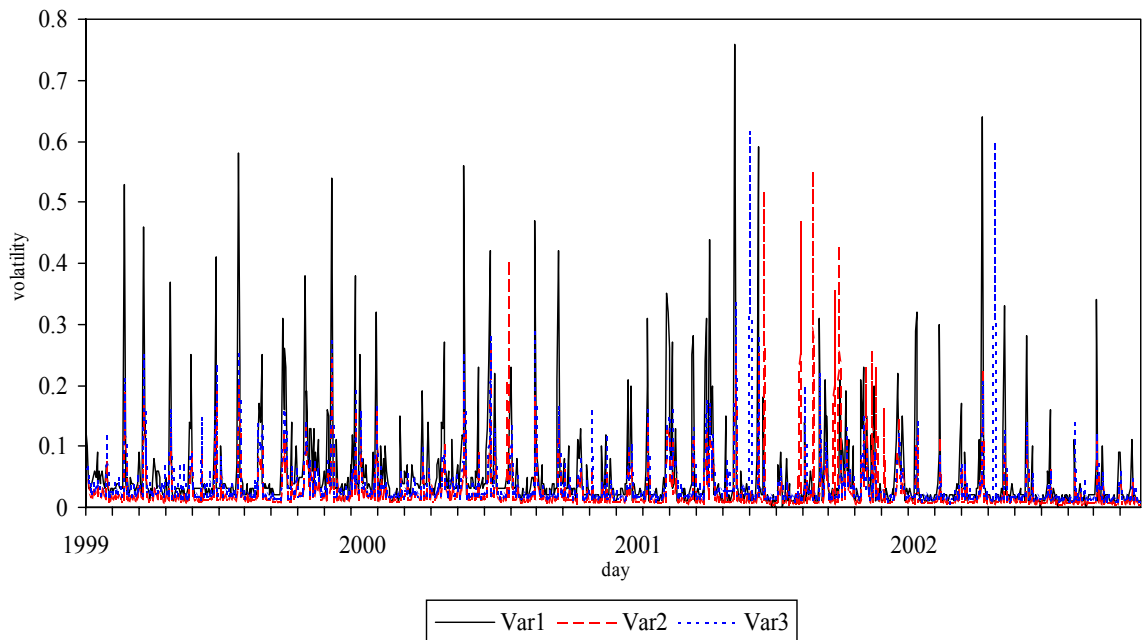
FIGURE 2  
Distances of the Kolmogorov-Smirnov tests



Note: The graph plots the multidimensional scaling representation of the distances of the Kolmogorov-Smirnov tests of equalities of the distribution functions of rates in different days of the maintenance period

FIGURE 3

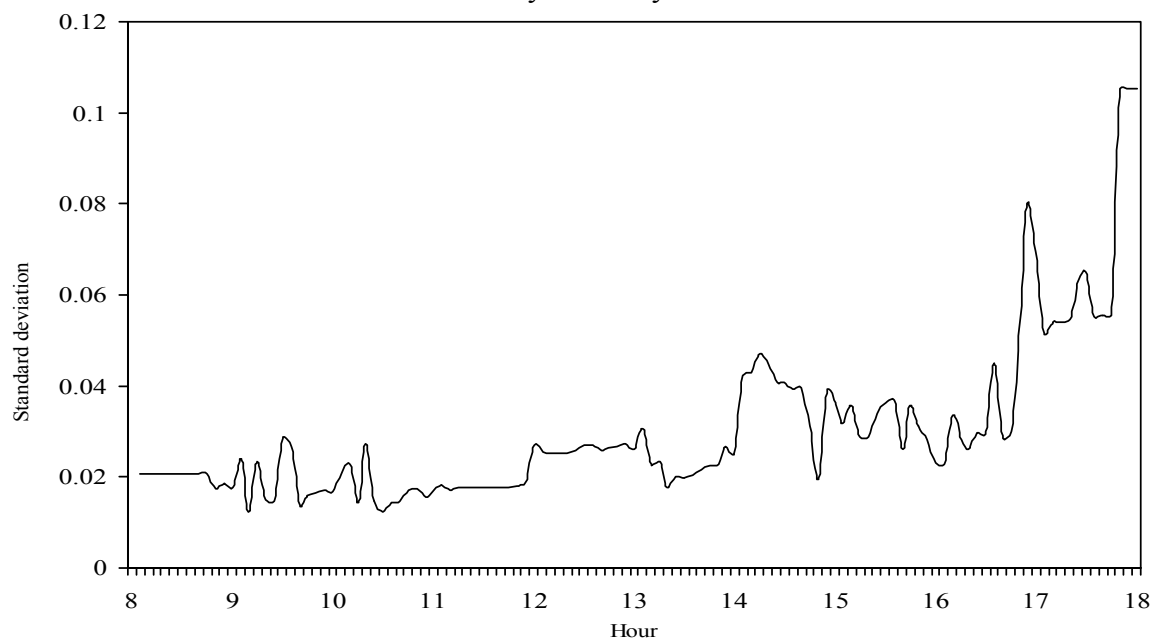
Different measures of dispersion of rates across banks



Note: Var1 is the difference between the rates paid by the Euro that represents the 10% cheaper and the 10% most expensive. Var2 is the standard deviation of rates weighted by the amount of Euro transacted by each bank and Var3 is the standard deviation of rates when each rate (each bank) is considered as one observation.

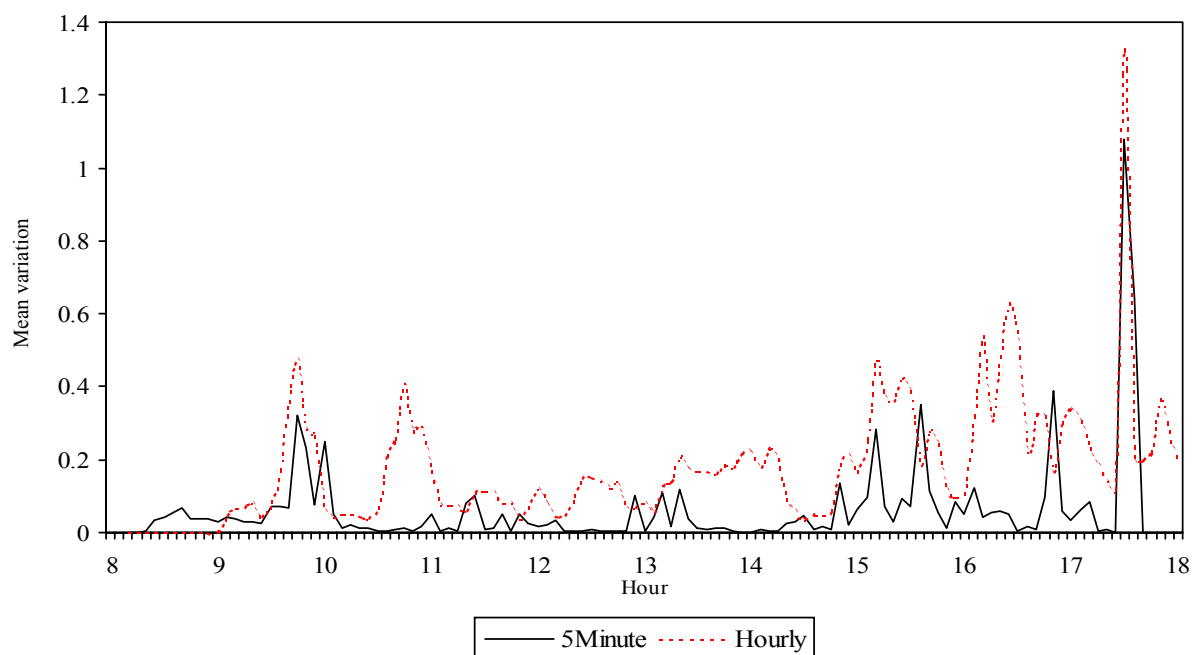
FIGURE 4

Intraday volatility of rates



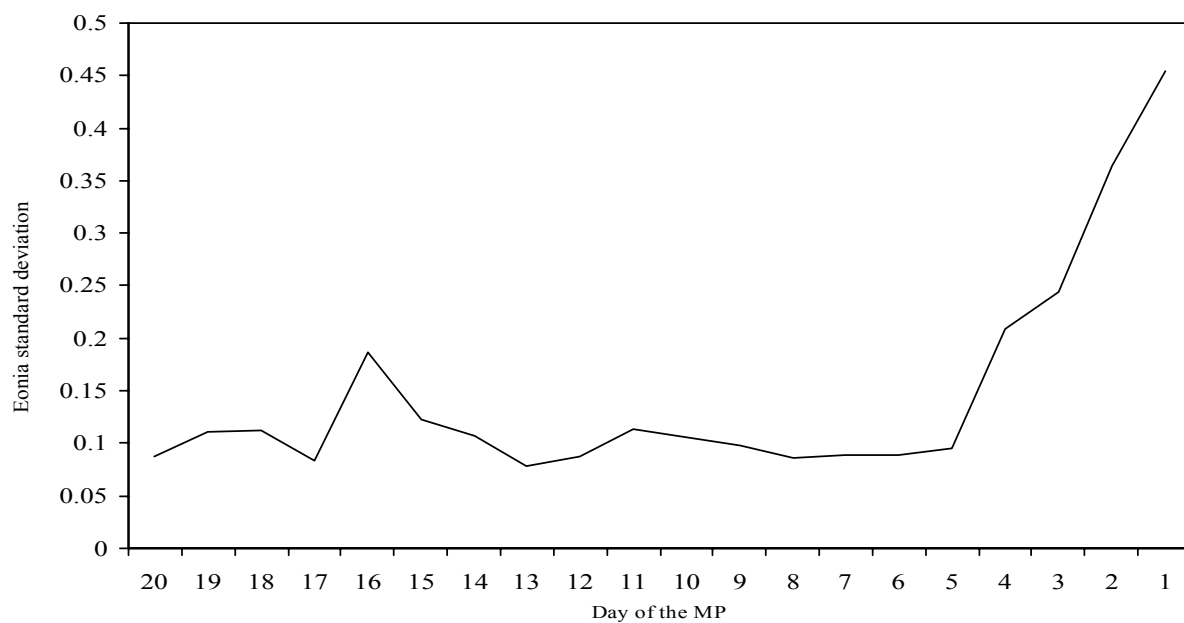
Note: The graph plots the average volatility of quotes for every 5 minute period interval for the emid data in the period March to September 2002.

FIGURE 5  
Intraday volatility of rates



The graph plots the average variation of the average rate quoted every five minutes with respect to the previous five minutes period and the previous hour. Emid data in the period March to September 2002.

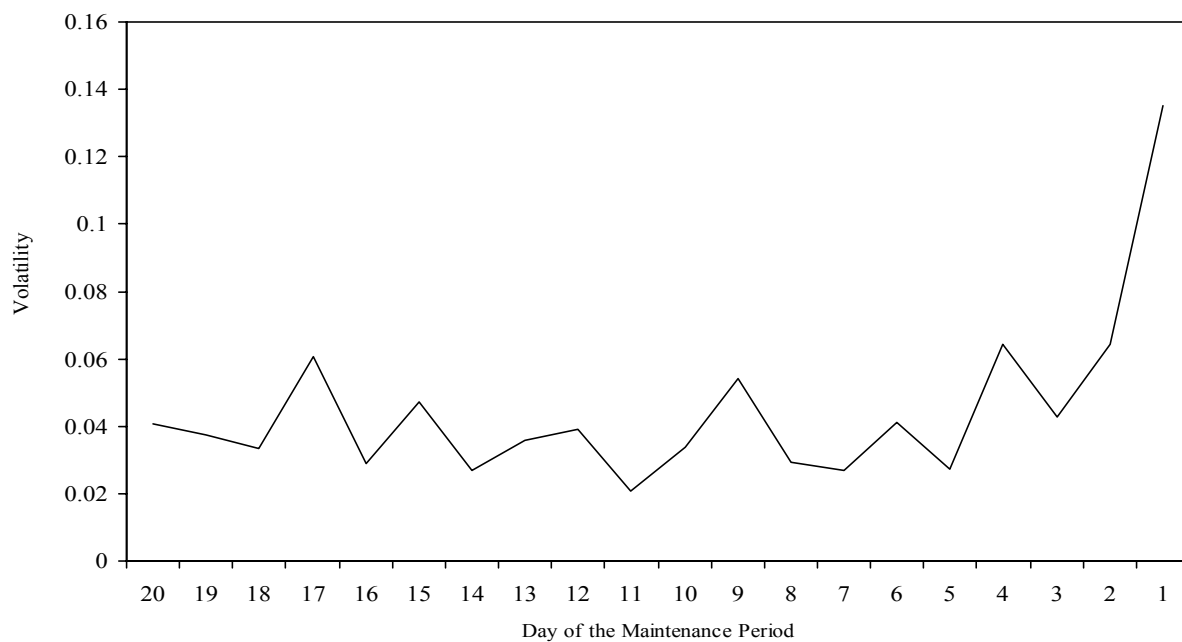
FIGURE 6  
Time series volatility of the Eonia



The graph plots the average absolute variation of Eonia rates in each day of the maintenance period

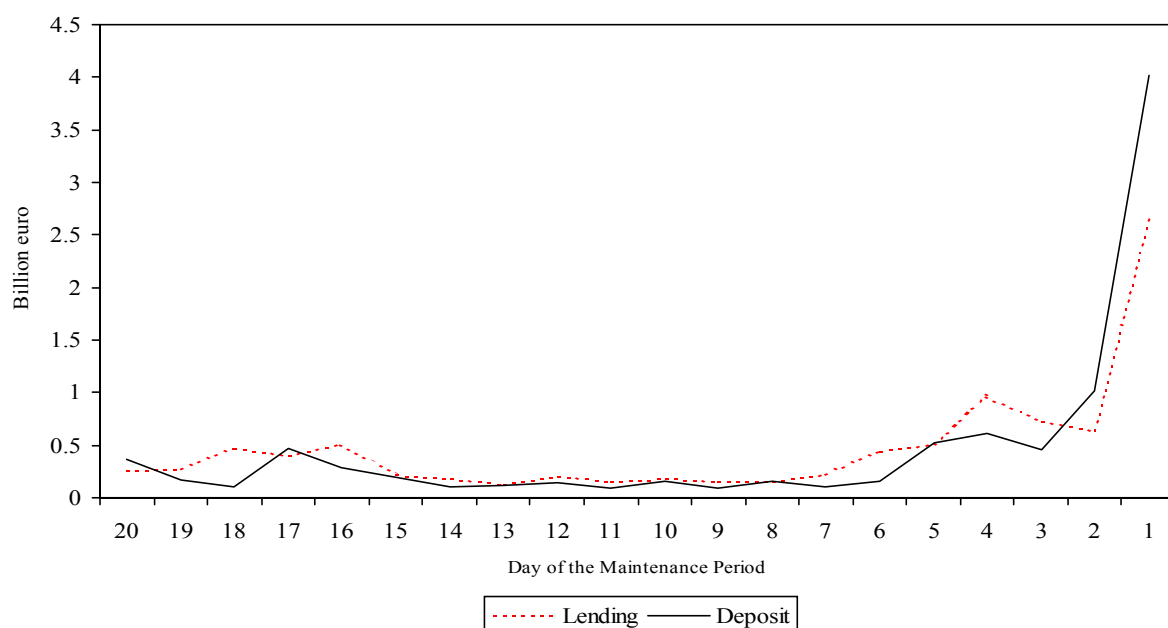


FIGURE 7  
Cross section volatility of the Eonia



The graph plots the average cross section standard deviation of Eonia rates in each day of the maintenance period

FIGURE 8  
Use of standing facilities



The graph plots the average use of the standing facilities in each day of the maintenance period